Calculation of CCT and Duv and Practical Conversion Formulae

Yoshi Ohno
Group Leader, NIST Fellow
Optical Technology Division
National Institute of Standards and Technology
Gaithersburg, Maryland  USA
White Light Chromaticity

CIE 1976 u'-v' diagram

Duv
CCT

Iso-CCT line: ±0.02 duv
Duv often missing

Lighting Facts Label

CCT and CRI do not tell the whole story of color quality
CCT and CRI do not tell the whole story

CIE 1976 $u'\text{-}v'$ diagram

- Duv = 0.015, Ra = 85.1
- Duv = 0.007, Ra = 85.5
- Duv = 0.001, Ra = 85.1

Not acceptable
Not preferred

Neodymium (Duv = -0.005)

Duv is another important dimension of chromaticity.
Duv defined in ANSI standard

Closest distance from the Planckian locus on the \((u', 2/3 \, v')\) diagram, with + sign for above and - sign for below the Planckian locus. (ANSI C78.377-2008)

Symbol: \(D_{uv}\)

CCT and Duv can specify the chromaticity of light sources just like \((x, y)\).

The two numbers (CCT, Duv) provides color information intuitively. \((x, y)\) does not.

Duv needs to be defined by CIE.
ANSI C78.377-2008 Specifications for the chromaticity of SSL products

CIE 1931 (x, y) Diagram

Table 1 - Nominal CCT Categories

<table>
<thead>
<tr>
<th>Nominal CCT (1))</th>
<th>Target CCT and tolerance (K)</th>
<th>Target Duv and tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2700 K</td>
<td>2725 ± 145</td>
<td>0.000 ± 0.006</td>
</tr>
<tr>
<td>3000 K</td>
<td>3045 ± 175</td>
<td>0.000 ± 0.006</td>
</tr>
<tr>
<td>3500 K</td>
<td>3465 ± 245</td>
<td>0.000 ± 0.006</td>
</tr>
<tr>
<td>4000 K</td>
<td>3985 ± 275</td>
<td>0.001 ± 0.006</td>
</tr>
<tr>
<td>4500 K</td>
<td>4503 ± 243</td>
<td>0.001 ± 0.006</td>
</tr>
<tr>
<td>5000 K</td>
<td>5028 ± 283</td>
<td>0.002 ± 0.006</td>
</tr>
<tr>
<td>5700 K</td>
<td>5665 ± 355</td>
<td>0.002 ± 0.006</td>
</tr>
<tr>
<td>6500 K</td>
<td>6530 ± 510</td>
<td>0.003 ± 0.006</td>
</tr>
<tr>
<td>Flexible CCT</td>
<td>(T^2) ± (\Delta T^3)</td>
<td>(D_{uv}^4) ± 0.006</td>
</tr>
</tbody>
</table>

1) Target CCT and tolerance values are rounded to the nearest integer.
2) \(T^2\) is the target CCT.
3) \(\Delta T^3\) is the tolerance range.
4) \(D_{uv}^4\) is the color difference metric.
Correlated Color Temperature (CCT)

Temperature [K] of a Planckian radiator whose chromaticity is closest to that of a given stimulus on the CIE \((u', 2/3 \cdot v')\) coordinate.

(CIE 15:2004)

CCT is based on the CIE 1960 \((u, v)\) diagram, which is now obsolete.

CCT is valid within distance 0.05 from the Planckian locus on the \((u', 2/3 \cdot v')\) diagram. (CIE 15: 2004)
APPENDIX E. INFORMATION ON THE USE OF PLANCK’S EQUATION FOR STANDARD AIR

According to the Planck’s law, the spectral radiance of a blackbody at thermodynamic temperature $T$ [K] in a medium having index of refraction $n$ is given by

$$L_{e,\lambda}(\lambda, T) = \frac{c_1 n^{-2} \lambda^{-5}}{\pi} \left[ \exp\left( \frac{c_2}{n\lambda T} \right) - 1 \right]^{-1}$$ (E.1)

where $c_1 = 2\pi hc^2$, $c_2 = hc/k$, $h$ is Planck’s constant, $c$ is the speed of light in vacuum, $k$ is the

$T$ should follow the current International Temperature Scale (ITS-90), therefore,

$c_2 = 1,4388 \times 10^{-2}$ m K.

Therefore, in the current recommendation in CIE 15:2004, colour temperature and correlated colour temperature are calculated using Eq. E.1 with $n = 1$ (exactly 1), thus no change from the previous practice. This recommendation may be subject to change in the future.

**Table III. Maximum errors of computed values of correlated color temperature, based on use of the 31 isotemperature lines listed in Table II.**

<table>
<thead>
<tr>
<th>µm</th>
<th>Range</th>
<th>K</th>
<th>Maximum error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>100 000-100 000</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>10-20</td>
<td>10 000-50 000</td>
<td>0.09</td>
<td>450</td>
</tr>
<tr>
<td>20-30</td>
<td>50 000-33 333</td>
<td>0.08</td>
<td>140</td>
</tr>
<tr>
<td>30-40</td>
<td>33 333-25 000</td>
<td>0.08</td>
<td>65</td>
</tr>
<tr>
<td>40-50</td>
<td>25 000-20 000</td>
<td>0.07</td>
<td>36</td>
</tr>
<tr>
<td>50-60</td>
<td>20 000-16 667</td>
<td>0.05</td>
<td>18</td>
</tr>
<tr>
<td>60-70</td>
<td>16 667-14 286</td>
<td>0.04</td>
<td>9.6</td>
</tr>
<tr>
<td>70-80</td>
<td>14 286-12 500</td>
<td>0.03</td>
<td>5.4</td>
</tr>
<tr>
<td>80-90</td>
<td>12 500-11 111</td>
<td>0.03</td>
<td>3.6</td>
</tr>
<tr>
<td>90-100</td>
<td>11 111-10 000</td>
<td>0.03</td>
<td>2.8</td>
</tr>
<tr>
<td>100-125</td>
<td>10 000-8 000</td>
<td>0.07</td>
<td>5.9</td>
</tr>
<tr>
<td>125-150</td>
<td>8 000-6 667</td>
<td>0.03</td>
<td>1.5</td>
</tr>
<tr>
<td>150-175</td>
<td>6 667-5 714</td>
<td>0.03</td>
<td>1.8</td>
</tr>
<tr>
<td>175-200</td>
<td>5 714-5 000</td>
<td>0.03</td>
<td>1.0</td>
</tr>
<tr>
<td>200-225</td>
<td>5 000-4 444</td>
<td>0.04</td>
<td>1.0</td>
</tr>
<tr>
<td>225-250</td>
<td>4 444-4 000</td>
<td>0.05</td>
<td>0.8</td>
</tr>
<tr>
<td>250-275</td>
<td>4 000-3 636</td>
<td>0.05</td>
<td>0.7</td>
</tr>
<tr>
<td>275-300</td>
<td>3 636-3 333</td>
<td>0.04</td>
<td>0.5</td>
</tr>
<tr>
<td>300-325</td>
<td>3 333-3 077</td>
<td>0.03</td>
<td>0.3</td>
</tr>
<tr>
<td>325-350</td>
<td>3 077-2 857</td>
<td>0.03</td>
<td>0.2</td>
</tr>
<tr>
<td>350-375</td>
<td>2 857-2 667</td>
<td>0.02</td>
<td>0.2</td>
</tr>
<tr>
<td>375-400</td>
<td>2 667-2 500</td>
<td>0.03</td>
<td>0.2</td>
</tr>
<tr>
<td>400-425</td>
<td>2 500-2 333</td>
<td>0.04</td>
<td>0.2</td>
</tr>
<tr>
<td>425-450</td>
<td>2 333-2 222</td>
<td>0.04</td>
<td>0.2</td>
</tr>
<tr>
<td>450-475</td>
<td>2 222-2 105</td>
<td>0.04</td>
<td>0.2</td>
</tr>
<tr>
<td>475-500</td>
<td>2 105-2 000</td>
<td>0.04</td>
<td>0.2</td>
</tr>
<tr>
<td>500-525</td>
<td>2 000-1 905</td>
<td>0.04</td>
<td>0.2</td>
</tr>
<tr>
<td>525-550</td>
<td>1 905-1 818</td>
<td>0.05</td>
<td>0.2</td>
</tr>
<tr>
<td>550-575</td>
<td>1 818-1 739</td>
<td>0.05</td>
<td>0.2</td>
</tr>
<tr>
<td>575-600</td>
<td>1 739-1 667</td>
<td>0.06</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Direct approach (1) to calculate CCT and Duv

Triangular solution

1) Create a table of CCT vs distance $d_i$ to BB locus on (u,v) coordinate.

2) Find the closest point in the table.

3) Solve the triangle for the neighboring 2 points

$$x = \frac{d_{m-1}^2 - d_{m+1}^2 + l^2}{2l}$$

$$T_x = T_{m-1} + (T_{m+1} - T_{m-1}) \cdot \frac{x}{l}$$

$$D_{uv} = \pm \text{sign} \left( d_{m-1}^2 - x^2 \right)^{1/2}$$

Use Planck’s equation and color matching functions at 1 nm interval.
CCT Error in Triangular Solution

Error increases when the point is far from Planckian locus.
Direct approach (2) to calculate CCT and Duv

Parabolic solution

(1) Create a table of CCT vs distance $d_i$ to BB locus on $(u,v)$ coordinate.

(2) Find the closest point in the table.

(3) Parabolic fit for the neighboring 3 points.

$$d(T) = aT^2 + bT + C$$

$$d(T)' = 2aT_x + b = 0 \quad \therefore T_x = \frac{-b}{2a}$$

$$D_{uv} = [\pm \text{sign}] \left( aT_x^2 + bT_x + C \right)$$
Much better, but the problem is on or very close to Planckian locus.
Use Parabolic solution but, take the CCT of Triangular solution for
\[ |D_{uv}| < 0.002 \]
Most Accurate Version (cascade expansion)

Used as the reference for accuracy verification.
Conversion from (CCT, Duv) back to (x, y)

Input: CCT $T$ (K)
       Duv $D_{uv}$

1) Calculate $(u_0, v_0)$ of the Planckian radiator at $T$ (K).
2) Calculate $(u_1, u_1)$ of the Planckian radiator at $T + 0.01$ (K).
3) Calculate

\[
\begin{align*}
\text{du} &= u_1 - u_0 \\
\text{dv} &= v_1 - v_0 \\
u &= u_0 + D_{uv} \cdot \sin \theta \\
&= u_0 + D_{uv} \cdot \frac{dv}{\sqrt{du^2 + dv^2}} \\
v &= v_0 + D_{uv} \cdot \cos \theta \\
&= v_0 + D_{uv} \cdot \frac{du}{\sqrt{du^2 + dv^2}}
\end{align*}
\]

\[u' = u, \quad v' = 1.5v, \quad x = \frac{9u'/(6u'-16v'+12)}{y = \frac{2v'/(3u'-8v'+6)}}\]

(Included in Revision draft of C78.377)
Accuracy of Most Accurate Version (4 stage)

Accuracy verification for 4-stage version

<table>
<thead>
<tr>
<th>CCT</th>
<th>2900</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duv</td>
<td>0.0200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>0.478420</th>
<th>Output</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0.473737</td>
<td></td>
<td></td>
</tr>
<tr>
<td>u</td>
<td>0.247629</td>
<td></td>
<td></td>
</tr>
<tr>
<td>v</td>
<td>0.367808</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

CCT 4-stage version
Simple calculation from \((x,y)\) or \((u',v')\) to \(D_{uv}\)

\(D_{uv}\) is normally calculated in the process of calculating CCT. Below is a simple approximation formula, without calculation of CCT.

1) Convert \((x, y)\) or \((u', v')\) to \((u, v)\)

\[
u = \frac{4x}{-2x + 12y + 3} \quad \text{or} \quad u = u' \\
v = \frac{6y}{-2x + 12y + 3} \quad \text{or} \quad v = \frac{2v'}{3}
\]

2) \(D_{uv}\) is obtained by

\[
I_{TP} = \sqrt{(u - 0.292)^2 + (v - 0.24)^2} \\
a = \arccos\left(\frac{u - 0.292}{I_{TP}}\right) \\
I_{BB} = k_6 a^6 + k_5 a^5 + k_4 a^4 + k_3 a^3 + k_2 a^2 + k_1 a + k_0 \\
D_{uv} = I_{TP} - I_{BB}
\]

(Include in Revision draft of C78.377)
Simple calculation from \((x,y)\) or \((u',v')\) to \(D_{uv}\)

Accuracy of this method

within 0.00001 in the range from 2600 K to 20000 K and \(D_{uv} 0.000 \pm 0.010\)

within 0.0001 in the range from 2160 K to 20000 K and \(D_{uv} 0.000 \pm 0.010\)

(Included in Revision draft of C78.377)
Simple calculation from \((x,y)\) or \((u',v')\) to \((CCT, Duv)\)

\[
I_{TP} = \sqrt{(u - 0.292)^2 + (v - 0.24)^2}
\]

\[
a_1 = \arctan((v - 0.24)/(u - 0.292)), \text{ if } a_1 \geq 0, \quad a = a_1; \quad \text{if } a_1 < 0, \quad a = a_1 + \pi
\]

\[
I_{BB} = k_6 a^6 + k_{05} a^5 + k_{04} a^4 + k_{03} a^3 + k_{02} a^2 + k_{01} a + k_{00}
\]

\[
D_{uv} = I_{TP} - I_{BB}
\]

For \(a < 2.54; \quad T_1 = 1/(k_{16} \cdot a^6 + k_{15} \cdot a^5 + k_{14} \cdot a^4 + k_{13} \cdot a^3 + k_{12} \cdot a^2 + k_{11} \cdot a + k_{10})
\]

For \(a \geq 2.54; \quad T_1 = 1/(k_{26} \cdot a^6 + k_{25} \cdot a^5 + k_{24} \cdot a^4 + k_{23} \cdot a^3 + k_{22} \cdot a^2 + k_{21} \cdot a + k_{20})
\]

For \(a < 2.54; \quad \Delta T_{c1} = (k_{36} \cdot a^6 + k_{35} \cdot a^5 + k_{34} \cdot a^4 + k_{33} \cdot a^3 + k_{32} \cdot a^2 + k_{31} \cdot a + k_{30}) \cdot (I_{BB} + 0.01)/L_p \cdot D_{uv}/0.01
\]

For \(a \geq 2.54; \quad \Delta T_{c1} = 1/(k_{46} \cdot a^6 + k_{45} \cdot a^5 + k_{44} \cdot a^4 + k_{43} \cdot a^3 + k_{42} \cdot a^2 + k_{41} \cdot a + k_{40}) \cdot (I_{BB} + 0.01)/L_p \cdot D_{uv}/0.01
\]

\[
T_2 = T_1 - \Delta T_{c1}, \quad c = \log(T_2)
\]

For \(Duv \geq 0; \quad \Delta T_{c2} = (k_{56} \cdot c^6 + k_{55} \cdot c^5 + k_{54} \cdot c^4 + k_{53} \cdot c^3 + k_{52} \cdot c^2 + k_{51} \cdot c + k_{50})
\]

For \(Duv < 0; \quad \Delta T_{c2} = (k_{66} \cdot c^6 + k_{65} \cdot c^5 + k_{64} \cdot c^4 + k_{63} \cdot c^3 + k_{62} \cdot c^2 + k_{61} \cdot c + k_{60}) \cdot |D_{uv}/0.03|^2
\]

\[
T_{FINAL} = T_2 - \Delta T_{c2}
\]

<table>
<thead>
<tr>
<th>(i)</th>
<th>(k6)</th>
<th>(k5)</th>
<th>(k4)</th>
<th>(k3)</th>
<th>(k2)</th>
<th>(k1)</th>
<th>(k0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-3.7146000E-03</td>
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<td>-3.307009E-01</td>
<td>9.750013E-01</td>
<td>-1.5008606E+00</td>
<td>1.115559E+00</td>
<td>-1.77348E-01</td>
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<tr>
<td>1</td>
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<td>8.2.7141290956E+09</td>
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<td>-1.9500061E+05</td>
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<tr>
<td>6</td>
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<td>1.4101538E+05</td>
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<td>2.48526954E+06</td>
<td>-4.11436958E+06</td>
<td>2.8151771E+06</td>
</tr>
</tbody>
</table>
Simple calculation from \((x,y)\) or \((u',v')\) to \((CCT, Duv)\)

Accuracy of this method
Conclusions

- Practical calculation and conversion formulae for CCT and Duv have been developed.
- Accuracies of some of the formulae will be further improved.
- The use of CCT and Duv (rather than $x$, $y$ or $u'$, $v'$ chromaticity coordinates) is recommended to specify the chromaticity of lighting sources.
THANK YOU for your attention.

Contact: ohno@nist.gov
Proposed revision of ANSI C78.377

- All center points to be moved onto the Planckian locus.
- This proposal is pending due to a need for vision experiments.
  - Anecdotes say people prefer below the Planckian locus.
  - NIST is funded by DOE to conduct vision experiments using STLF.
CCT (K) difference between 3rd stage and 5th stage
Summary

- Duv is important for color quality of light sources.
- Duv is often neglected in specifications.
- Parabolic and triangle combined solution works well for CCT calculation.
- 1 % step table provides enough accuracy
  (<1 K for 1000 to 10000 K, <2 K up to 20000 K, Duv± 0.03)
- Most Accurate Version (cascade expansion),
- Conversion from (CCT, Duv) back to (x,y),
- Simple calculation from (x,y) or (u′,v′) to Duv,
- Simple calculation from (x,y) or (u′,v′) to (CCT, Duv) have been developed.